Unit = I

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~~Softwar~~ Solution of Polynomial transcendental equations :-

Polynomial / algebraic equations

An expression of the form

$$P_n(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1} x + a_n$$

Where $a_0, a_1, \ldots a_n$ are constants.

n is posi integer

is called a polynomial in x of degree n

provided $a_0 \neq 0$

Ex $f(x) = x^3 - 4x - 3 = 0$

Transcendental Equation

If a function $f(x)$ contains trignometric, logarithm, exponential etc functions,

then $f(x) = 0$ is a called transcendental equations

Ex $f(x) = 3x - \cos x - 1$

Methods to solve Polynomial & transcendental equations.

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Example 2 - Newton Raphson Method

Lecture - 4

Q.2 Using Newton's iterative method, find the real root of $x \log_{10} x = 1.2$, correct to five decimal places RGPV (2010, 2005)

Solution :-

$$f(x) = x \log_{10} x - 1.2$$

$$f(0) = -1.2 \quad (-ve)$$

$$f(1) = -1.2 \quad (-ve)$$

$$f(2) = 2 \log_{10} 2 - 1.2 = -0.59794 \quad (-ve)$$

Sign Change

$$f(3) = 3 \log_{10} 3 - 1.2 = +1.4314 - 1.2 = 0.23136$$

(+ve)

So, a root of $f(x) = 0$ lies between 2 and 3

let us take

$$x_0 = 2$$

$$f'(x) = \log_{10} x + x \cdot \frac{1}{x} \log_{10} e$$

$$= \log_{10} x + 0.43429$$

Lecture 23

Q Given the values

x	5	7	11	13	17
$f(x)$	150	392	1492	2366	5302

Evalute $f(9)$ using Newton's divided difference formula :-

Solution the divided difference table as.

x	y	I $[x_0, x_1]$	II $[x_0, x_1, x_2]$	III $[x_0, x_1, x_2, x_3]$
5	150			
		$\frac{392-150}{7-5} = 121$		
7	392		$\frac{265-121}{11-5} = 24$	
		$\frac{1492-392}{11-7} = 265$		$\frac{32-24}{13-5} = 1$
11	1492		$\frac{457-265}{13-7} = 32$	
		$\frac{2366-1492}{13-11} = 457$		$\frac{42-32}{17-7} = 1$
13	2366		$\frac{705-457}{17-11} = 42$	
		$\frac{5302-2366}{17-13} = 709$		
17	5302			

If the Probability that an individual offers a bad reaction from a certain infection is 0.001 determine the probability that out of 2000 individuals.

(i) exact 3

(ii) more than 2 individuals $x>2$

(iii) none

(iv) more than 1 individuals $x>1$

will suffer a bad reaction

given

$P = 0.001 \qquad n = 2000$

$m = nP = 2000 \times 0.001$

$= 2$

using poisson's formula

$$P(x) = \frac{e^{-m} m^x}{x!}$$

(i) $x = 3$

$$P(x=3) = \frac{e^{-2}(2)^3}{3!} = 0.18$$

$(\because \Sigma P = 1)$

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Sol

$\lambda = 1$

$$P(x > 3) = \int_3^{\infty} f(x)\, dx$$

$$= \int_3^{\infty} \lambda e^{-\lambda x}\, dx$$

$$= \int_3^{\infty} e^{-x}\, dx$$

$$= \left[\frac{e^{-x}}{-1}\right]_3^{\infty}$$

$$= -\left[e^{-\infty} - e^{-3}\right]$$

$$= -\left[\frac{1}{e^{\infty}} - \frac{1}{e^{3}}\right]$$

$$= -\left[\frac{1}{\infty} - \frac{1}{e^{3}}\right]$$

$$= -\left[0 - \frac{1}{e^{3}}\right]$$

$$= 1/e^{3}$$

Gauss's elimination method

$$x_1 - 2x_2 - 6x_3 = 12$$

$$2x_1 - 4x_2 + 12x_3 = -17$$

$$x_1 - 4x_2 - 12x_3 = 22$$

Sol Augmented matrix

$$\begin{array}{c} \left[\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 2 & 4 & 12 & -17 \\ 1 & -4 & -12 & 22 \end{array}\right] \end{array} \begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}$$

$$R_2 \to R_2 \to 2R_1$$

$$R_3 \to R_3 \to R_1$$

$$\left[\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 0 & 8 & 24 & -41 \\ 0 & -2 & -6 & 10 \end{array}\right]$$

$$R_3 \leftarrow R_3/2$$

$$\left[\begin{array}{cccc} 1 & -2 & -6 & 12 \\ 0 & 8 & 24 & -41 \\ 0 & -1 & -3 & 5 \end{array}\right]$$

$$R_3 \to 8R_3 + R_2$$

L-53

② Simpson's $\frac{1}{3}$ rule

better approximation

It is used when n is multiple of 2

$n = 2, 4, 6, 8 \ldots$

$$\int_a^b f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_0+y_3+y_5+\ldots) + 2(y_2+y_4+y_6+\ldots)\right]$$

③ Simpson $\frac{3}{8}$ rule

It is used when $\boxed{n \text{ is multiple of } 3\ (3, 6, 9, \ldots)}$

$$\int_a^b f(x)\,dx = \frac{3h}{8}\left[(y_0+y_n) + 3(y_1+y_2+y_4+\ldots) + 2\underbrace{(y_3+y_6+y_9+\ldots)}_{\text{multiple of } 3}\right]$$

L=70

substutut in eqⁿ ①

$$= (-1)(-1)\frac{(s^2-a^2)}{(s^2+a^2)^2}$$

$$= \frac{s^2-a^2}{(s^2+a^2)^2}$$ Ans

$L\{t^2 \sin at\}$

Sol $$L\{\sin at\} = \frac{a}{s^2+a^2}$$

$$L\{t^2 \sin at\} = (-1)^2 \frac{d^2}{ds^2}\left(\frac{a}{s^2+a^2}\right)$$

$$= a\frac{d}{ds}\left(\frac{d}{ds}(s^2-a^2)^{-1}\right)$$

$$= a\frac{d}{ds}\left((-1)(s^2+a^2)^{-2} \times 2s\right.$$

$$= -2a \frac{d}{ds} \frac{s}{(s^2+a^2)^2} \quad ①$$

Now $$\frac{d}{ds} \frac{s}{(s^2+a^2)} \quad \frac{u}{v}$$

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Application of Laplace Transform

Solving Differential Equations using Laplace Transform

Given

First order defference eq^n

$$\frac{dx}{dt} + x = 1$$

Or second order differential eq^n

$$\frac{d^2x}{dt^2} + \frac{6dx}{dt} + 9x = 50 \sin t$$

third forth

& some conditions $x=0$ when $t=0$

Or $x(0) = 0$

$$\frac{dx}{dt} = 0 \text{ or } x'(0) = 0$$

step 1 calculat laplace transform of derivatives using formula

$$L\{f^n(t)\} = s^n L\{f(t)\} - s^{n-1} f(0) - s^{n-2} f'(0) - s^{n-3} f''(0) \ldots$$